## $0^+ \to 2^+ \ 0 \nu \beta \beta$ decay triggered directly by the Majorana neutrino mass

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## Abstract

We treat  $0^+ \to 2^+ \ 0\nu\beta\beta$  decays taking into account recoil corrections to the nuclear currents. The decay probability can be written as a quadratic form of the effective coupling constants of the right-handed leptonic currents and the effective neutrino mass. We calculate the nuclear matrix elements for the  $0^+ \to 2_1^+ \ 0\nu\beta\beta$  decays of  $^{76}$ Ge and  $^{100}$ Mo, and demonstrate that the relative sensitivities of  $0^+ \to 2^+$  decays to the neutrino mass and the right-handed currents are comparable to those of  $0^+ \to 0^+$  decays.

The neutrinoless double beta  $(0\nu\beta\beta)$  decay can take place through an exchange of neutrino between two quarks in nulei if the electron neutrino is a Majorana particle and has a nonvanishing mass and/or right-handed couplings [1–3]. There may be other possible mechanisms such as those involving supersymmetric particles which also cause the decay of two neutrons into two protons and two electrons [4–6]. In the present work, however, we restrict ourselves to the conventional two-nucleon and  $\Delta$  mechanisms of  $0\nu\beta\beta$  decay through light Majorana neutrino exchange. From the analises of experimental data on  $0^+ \to 0^+ \ 0\nu\beta\beta$  decays, stringent limits on the effective neutrino mass and the effective coupling constants of the right-handed leptonic currents have been deduced (see e.g. [3,7] and the references quoted therein). On the other hand it still seems to be believed widely that  $0^+ \to 2^+ 0\nu\beta\beta$  decays are sensitive only to the right-handed currents. In view of the theorem that the electron neutrino should have a nonvanishing Majorana mass if  $0\nu\beta\beta$  decay occurs anyway [8–10], an observation of  $0\nu\beta\beta$  decay due to right-handed interactions would certainly mean also a nonvanishing Majorana mass of the electron neutrino. The purpose of the present work is, however, not to investigate the role of the

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Majorana neutrino mass in such a sense, but to demonstrate that it causes  $0^+ \to 2^+ \ 0\nu\beta\beta$  decays directly.

A direct contribution of the neutrino mass to  $0^+ \to 2^+ 0\nu\beta\beta$  decays was considered in [3] taking into account the nuclear recoil currents, and the inverse half-life was given as

$$\left[\tau_{1/2}^{0\nu}(0^+ \to 2^+)\right]^{-1} = F_{1+}(Z_{1+})^2 + F_{1-}(Z_{1-})^2 + F_{2+}(Z_{2+})^2 + F_{2-}(Z_{2-})^2, \tag{1}$$

where  $F_{j\pm}$  (j=1,2) are the phase space integrals and

$$Z_{1\pm} = M_{\lambda} \langle \lambda \rangle - M_{\eta} \langle \eta \rangle \pm M_{\rm m} \frac{\langle m_{\nu} \rangle}{m_{\rm e}},$$

$$Z_{2\pm} = M_{\eta}' \langle \eta \rangle \pm M_{\rm m} \frac{\langle m_{\nu} \rangle}{m_{\rm e}},$$
(2)

with the electron mass  $m_{\rm e}$  and

$$\langle m_{\nu} \rangle = \sum_{j}^{\prime} U_{ej}^{2} m_{j} ,$$

$$\langle \lambda \rangle = \lambda \sum_{j}^{\prime} U_{ej} V_{ej} ,$$

$$\langle \eta \rangle = \eta \sum_{j}^{\prime} U_{ej} V_{ej} .$$
(3)

Here  $m_j$  is the mass of the eigenstate Majorana neutrino  $N_j$ .  $U_{ej}$  and  $V_{ej}$  are the amplitudes of  $N_j$  in the left- and right-handed electron neutrinos,  $\lambda$  and  $\eta$  the coupling constants of the right-handed leptonic current with the right-and left-handed hadronic currents, and the summations should be taken over light neutrinos ( $m_j \ll 100 \text{ MeV}$ ). The nuclear matrix elements  $M_{\alpha}$  ( $\alpha = \lambda, \eta$ , m) are defined by

$$M_{\alpha} = \langle 2_{\rm F}^{+} \| \frac{1}{2} \sum_{n,m} \tau_{n}^{+} \tau_{m}^{+} (\boldsymbol{M}_{\alpha})_{nm} \| 0_{\rm I}^{+} \rangle .$$
 (4)

The explicit forms of the two body operators  $M_{\lambda}$ ,  $M_{\eta}$  and  $M'_{\eta}$  were given in [11] including the contribution of the  $\Delta$  mechanism, in which the  $0\nu\beta\beta$  decay proceed through an exchange of a Majorana neutrino between two quarks in the same baryon in a nucleus. On the other hand the operator  $M_{\rm m}$  was derived in [3] as

$$(\boldsymbol{M}_{\mathrm{m}})_{nm} = -\frac{1}{2}im_{\mathrm{e}} \{ [\boldsymbol{r}_{nm} \otimes (\boldsymbol{\sigma}_{n}C_{m} - \boldsymbol{\sigma}_{m}C_{n})]^{(2)} \}$$

$$+i(g_{V}/g_{A})[\boldsymbol{r}_{nm}\otimes(\boldsymbol{D}_{n}\times\boldsymbol{\sigma}_{m}-\boldsymbol{D}_{m}\times\boldsymbol{\sigma}_{n})]^{(2)} +(g_{V}/g_{A})^{2}[\boldsymbol{r}_{nm}\otimes(\boldsymbol{D}_{n}-\boldsymbol{D}_{m})]^{(2)}\}H(r_{nm}),$$
(5)

where  $\mathbf{r}_{nm} = \mathbf{r}_n - \mathbf{r}_m$ , H(r) is the neutrino propagation function,  $g_{\rm V}$  and  $g_{\rm A}$  the vector and axial vector coupling constants.  $C_n$  and  $\mathbf{D}_n$  are the recoil correction terms to the axial vector and vector nuclear currents [2,12] given by

$$C_{n} = (\boldsymbol{p}_{n} + \boldsymbol{p}_{n}') \cdot \boldsymbol{\sigma}_{n}/2M,$$

$$\boldsymbol{D}_{n} = [\boldsymbol{p}_{n} + \boldsymbol{p}_{n}' - i\mu_{\beta}\boldsymbol{\sigma}_{n} \times (\boldsymbol{p}_{n} - \boldsymbol{p}_{n}')]/2M,$$
(6)

where  $\boldsymbol{p}_n$  and  $\boldsymbol{p}'_n$  are the initial and final nucleon momenta, M the nucleon mass, and  $\mu_{\beta}=4.7$ . The above expression for  $\boldsymbol{M}_{\mathrm{m}}$  is, however, not suitable for numerical calculations as it stands. Therefore, as was done for  $\boldsymbol{M}_{\lambda}$ ,  $\boldsymbol{M}_{\eta}$  and  $\boldsymbol{M}'_{\eta}$  in [11], we expand it in terms of the operators  $\boldsymbol{M}_{inm}$  with simpler spin and orbital structures,

$$(\boldsymbol{M}_{\mathrm{m}})_{nm} = \sum_{i} C_{\mathrm{m}i} \boldsymbol{M}_{inm} . \tag{7}$$

We define the matrix element  $M_i$  of the operator  $\mathbf{M}_{inm}$  analogously to Eq. (4). The coefficients  $C_{mi}$  and the two-body operators  $\mathbf{M}_{inm}$  are listed in Table 1, where

$$h = r_{nm}H(r_{nm}), \qquad h' = -r_{nm}H'(r_{nm}),$$

$$S_{\lambda nm} = [\boldsymbol{\sigma}_{n} \otimes \boldsymbol{\sigma}_{m}]^{(\lambda)}, \qquad S_{\pm nm} = \boldsymbol{\sigma}_{n} \pm \boldsymbol{\sigma}_{m},$$

$$\boldsymbol{y}_{Knm} = [\hat{\boldsymbol{r}}_{nm} \otimes \hat{\boldsymbol{r}}_{nm}]^{(K)}, \qquad \boldsymbol{Y}_{Knm} = [\hat{\boldsymbol{r}}_{nm} \otimes \hat{\boldsymbol{r}}_{+nm}]^{(K)} (r_{+nm}/r_{nm}),$$

$$\boldsymbol{y}'_{Knm} = i[\hat{\boldsymbol{r}}_{nm} \otimes \boldsymbol{p}_{nm}]^{(K)}, \qquad \boldsymbol{Y}'_{Knm} = i[\hat{\boldsymbol{r}}_{nm} \otimes \boldsymbol{P}_{nm}]^{(K)},$$

$$\boldsymbol{r}_{+nm} = \boldsymbol{r}_{n} + \boldsymbol{r}_{m}, \qquad \hat{\boldsymbol{a}} = \boldsymbol{a}/|\boldsymbol{a}|,$$

$$\boldsymbol{p}_{nm} = \frac{1}{2}(\boldsymbol{p}_{n} - \boldsymbol{p}_{m}), \qquad \boldsymbol{P}_{nm} = \boldsymbol{p}_{n} + \boldsymbol{p}_{m}. \qquad (8)$$

As was described in detail in [11],  $M_{\lambda}$  and  $M_{\eta}$  can be expanded in terms of  $M_{inm}$  with  $1 \leq i \leq 5$ ,  $8 \leq i \leq 13$ , and  $M'_{\eta}$  in terms of  $M_{inm}$  with i = 6, 7 (for the definition of  $M_{inm}$  with  $6 \leq i \leq 13$ , which do not appear in Table 1, see [11]). Of these operators,  $M_{inm}$  with  $8 \leq i \leq 13$  are related to the  $0\nu\beta\beta$  transitions which involve virtual  $\Delta$  particles in nuclei, and they are induced by the operator  $M_{2nm}$  interpreted as acting on two quarks in a nucleon or a  $\Delta$  particle.

The new operators  $M_{inm}$  with  $14 \le i \le 25$  appear only in the expansion of  $M_{\rm m}$ . In the derivation of  $C_{\rm m}$  listed in Table 1, we have not taken into ac-

count the  $\Delta$  mechanism yet. Under the same assumption of the non-relativistic constituent quark model about the  $\Delta$  mechanism as was made in [11], the operators in Table 1 except  $\boldsymbol{M}_{inm}$  with i=2,16,17 do not contribute when interpreted as acting on two quarks in a nucleon or a  $\Delta$  particle. Since the relation  $\mu_{\beta} = g_{\rm V} = g_{\rm A} = 1$  holds for the quark currents, we see  $C_{\rm m}i=0$  for i=2,16. The only possible contribution of  $\boldsymbol{M}_{17nm}$  to  $\boldsymbol{M}_{\rm m}$  is estimated to be about  $m_{\rm e}/2M$  of the  $\Delta$  mechanism contributions to  $\boldsymbol{M}_{\lambda}$  and  $\boldsymbol{M}_{\eta}$ . Therefore we will neglect the  $\Delta$  mechanism for the calculation of  $\boldsymbol{M}_{\rm m}$  in the present work.

We calculate the nuclear matrix elements for  $0_1^+ \to 2_1^+ \ 0\nu\beta\beta$  decay of <sup>76</sup>Ge and <sup>100</sup>Mo using the method given in [11]. We describe the initial  $0_1^+$  and final  $2_1^+$  nuclear states in terms of the Hartree-Fock-Bogoliubov type wave functions which are obtained by variation after particle-number and angular-momentum projection [11–13]. For the case of <sup>76</sup>Ge decay, the calculation of the matrix elements  $M_i$  with  $1 \le i \le 13$  has been performed in [11]. In the present work we calculate only the new ones with  $14 \le i \le 25$  using the nuclear wave functions obtained in [11]. In order to calculate all  $M_i$  with  $1 \le i \le 25$  for the <sup>100</sup>Mo decay, the nuclear wave functions of <sup>100</sup>Mo( $0_1^+$ ) and <sup>100</sup>Ru( $2_1^+$ ) are constructed in the same manner as in the case of the <sup>76</sup>Ge decay. Table 2 shows the calculated matrix elements  $M_{\rm m}$  for the <sup>76</sup>Ge and <sup>100</sup>Mo decays as a sum of the products  $C_{\rm m}iM_i$ . It should be noted that the matrix elements of the operators with rank 0 spin part, *i.e.*  $M_1$ ,  $M_4$ ,  $M_{14}$  and  $M_{15}$  have the dominant contributions to  $M_{\rm m}$ . Table 3 summarizes the calculated matrix elements  $M_{\lambda}$ ,  $M_{\eta}$ ,  $M'_{\eta}$  and  $M_{\rm m}$  for the <sup>76</sup>Ge and <sup>100</sup>Mo decays.

The differential rate for  $0^+ \to 2^+ \ 0\nu\beta\beta$  decay with the energy of one of the emitted electrons  $\epsilon_1$  and the angle between the two electrons  $\theta_{12}$  can be written as

$$\frac{\mathrm{d}^2 W_{0\nu}}{\mathrm{d}\epsilon_1 \mathrm{d}\cos\theta_{12}} = a^{(0)}(\epsilon_1) + a^{(1)}(\epsilon_1) P_1(\cos\theta_{12}) + a^{(2)}(\epsilon_1) P_2(\cos\theta_{12}). \tag{9}$$

Each of the angular correlation coefficients  $a^{(k)}(\epsilon_1)$  (k=0,1,2) can be expressed as a sum of the products of an electron phase space factor and a second order monomial of  $Z_{j\pm}$  defined in Eq. (2). The explicit form of  $a^{(0)}(\epsilon_1)$ , which yields  $(\ln 2)/2$  times the right hand side of Eq. (1) upon integration over  $\epsilon_1$ , can be readily obtained by combining the relevant equations in [3]. Since the expressions for  $a^{(1)}(\epsilon_1)$  and  $a^{(2)}(\epsilon_1)$  are rather complicated, they will be given elsewhere. Numerical calculations show that  $a^{(1)}(\epsilon_1)$  is dominated by a term with the factor  $-(Z_{1+})^2 + Z_{2+}Z_{2-}$  times a positive function of  $\epsilon_1$ , whereas  $a^{(2)}(\epsilon_1)$  by a term with the factor  $2Z_{1+}Z_{1-} - (Z_{2+})^2 - (Z_{2-})^2$ . For later reference we denote these two factors as  $z^{(1)}$  and  $z^{(2)}$ , respectively.

Figure 1 shows the single electron spectra  $dW_{0\nu}/d\epsilon_1 = 2a^{(0)}$  and the ratios of

the angular correlation coefficients  $a^{(1)}/a^{(0)}$  and  $a^{(2)}/a^{(0)}$  for the three limiting cases, (a)  $\langle \lambda \rangle \neq 0$ , (b)  $\langle \eta \rangle \neq 0$  and (c)  $\langle m_{\nu} \rangle \neq 0$ . Since the coefficients  $a^{(k)}(\epsilon_1)$  depend on the parameters  $\langle \lambda \rangle$ ,  $\langle \eta \rangle$  and  $\langle m_{\nu} \rangle$  through  $Z_{j\pm}$ , the results shown in Fig. 1 are independent of nuclear models for the cases (a) and (c). We can also easily understand the signs of  $a^{(1)}$  and  $a^{(2)}$  from the relations  $z^{(1)} = -(M_{\lambda}\langle \lambda \rangle)^2$  and  $z^{(2)} = 2(M_{\lambda}\langle \lambda \rangle)^2$  for the case (a), and  $z^{(1)} = -2(M_m\langle m_{\nu}\rangle/m_e)^2$  and  $z^{(2)} = -4(M_m\langle m_{\nu}\rangle/m_e)^2$  for the case (c). On the other hand for the case (b), we obtain  $z^{(1)} = -(M_{\eta}\langle \eta \rangle)^2 + (M'_{\eta}\langle \eta \rangle)^2$  and  $z^{(2)} = 2(M_{\eta}\langle \eta \rangle)^2 - 2(M'_{\eta}\langle \eta \rangle)^2$ , and consequently a cancellation between the contributions of  $M_{\eta}$  and  $M'_{\eta}$  occurs when these are of comparable magnitudes. This is just the case for the <sup>100</sup>Mo decay, but not for the <sup>76</sup>Ge decay where  $M'_{\eta}$  is much smaller than  $M_{\eta}$  so that there is no significant difference between the cases (a) and (b) in the angular correlation. It should also be noted in Fig. 1 that the single electron spectra for all the three cases (a), (b) and (c) have approximately the same shape. This is in contrast with the  $0^+ \to 0^+$  decays where the spectrum for  $\langle \lambda \rangle \neq 0$  is very different from those for  $\langle m_{\nu} \rangle \neq 0$  or  $\langle \eta \rangle \neq 0$  [2,3].

Using the matrix elements in Table 3 and the phase space integrals  $F_{j\pm}$  calculated in [3], we can deduce from the experimental data  $\tau_{1/2}^{0\nu}(0^+ \to 2_1^+) > 8.2 \times 10^{23}$  yr (90% C.L.) [14] for the <sup>76</sup>Ge decay the constraints on the right-handed current couplings and the effective neutrino mass listed in Table 4. As for the <sup>100</sup>Mo decay, the Osaka group has obtained the limit  $\tau_{1/2}^{0\nu}(0^+ \to 2_1^+) > 1.4 \times 10^{22}$  yr (68% C.L.) [15] assuming  $\langle \lambda \rangle \neq 0$ . Because of the differences in the angular correlation as we see from Fig. 1, an analysis of the same raw experimental data might yield a half-life limit significantly different from the above value especially for the case  $\langle \eta \rangle \neq 0$ . However we assume here just the same half-life limit also for the cases  $\langle \eta \rangle \neq 0$  and  $\langle m_{\nu} \rangle \neq 0$  in order to compare the resulting constraints with those from the <sup>76</sup>Ge data.

The limits which can be deduced from the experimental bound  $\tau_{1/2}^{0\nu}(0^+ \to 0^+) > 5.7 \times 10^{25}$  yr (90% C.L.) [16] on the  $0^+ \to 0^+$  decay of <sup>76</sup>Ge using the nuclear matrix elements of [17] are  $|\langle\lambda\rangle| < 3.8 \times 10^{-7}$ ,  $|\langle\eta\rangle| < 2.2 \times 10^{-9}$  and  $|\langle m_{\nu}\rangle| < 0.19$  eV. Comparing these limits with those of Table 4, we notice the considerable difference in the absolute sensitivities between the  $0^+ \to 0^+$  and  $0^+ \to 2^+$  decays, which reflects the smaller Q-value as well as the higher electron partial waves associated with the latter. However, it should be stressed here that the relative sensitivities to  $\langle m_{\nu}\rangle$  and  $\langle\eta\rangle$  are comparable in both cases. In other words,  $\langle m_{\nu}\rangle = 1$  eV would give roughly the same decay rate as  $\langle\eta\rangle = 10^{-8}$  in the  $0^+ \to 2^+$  as well as in the  $0^+ \to 0^+$  decays. At the same time it should also be noted that the  $0^+ \to 2^+$  decay is relatively more sensitive to  $\langle\lambda\rangle$ .

In summary, we have calculated  $0^+ \to 2^+ 0\nu\beta\beta$  decay rates taking into account the recoil corrections to the nuclear currents. As a result, the expression for the decay probability becomes a quadratic form of not only the effective

coupling constants  $\langle \lambda \rangle$  and  $\langle \eta \rangle$  of the right-handed leptonic currents but also the effective neutrino mass  $\langle m_{\nu} \rangle$  which would be totally absent without the inclusion of the recoil corrections. In other words, the recoil corrections give the *lowest* order contribution to the  $0^+ \to 2^+ 0\nu\beta\beta$  decay for the case where  $\langle \lambda \rangle = \langle \eta \rangle = 0$  and  $\langle m_{\nu} \rangle \neq 0$ . Furthermore, by the numerical calculation of the relevant nuclear matrix elements, we have demonstrated that the *relative* sensitivities of  $0^+ \to 2^+$  decays to  $\langle m_{\nu} \rangle$  and  $\langle \eta \rangle$  are comparable to those of  $0^+ \to 0^+$  decays.

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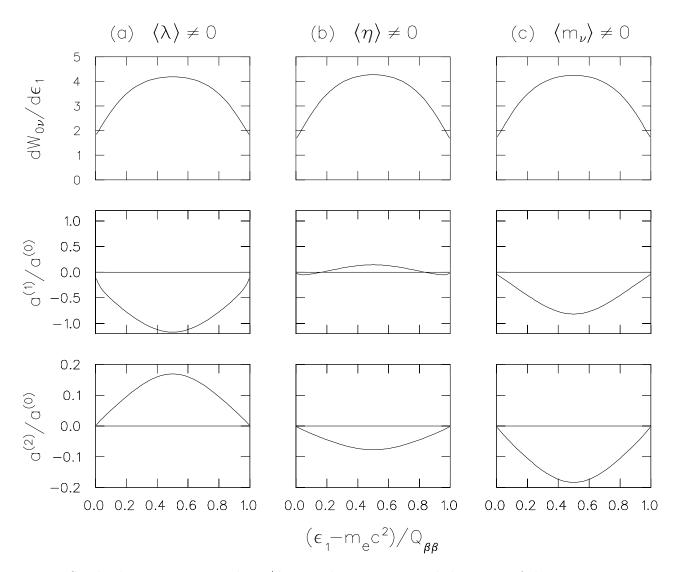


Fig. 1. Single electron spectrum  $\mathrm{d}W_{0\nu}/\mathrm{d}\epsilon_1$  in arbitrary units and the ratios of the angular correlation coefficients  $a^{(1)}/a^{(0)}$  and  $a^{(2)}/a^{(0)}$  for the  $0^+ \to 2_1^+ \ 0\nu\beta\beta$  decay of  $^{100}\mathrm{Mo}$ . They are all plotted against the kinetic energy fraction of one of the two emitted electrons, where  $Q_{\beta\beta}(0^+ \to 2_1^+) = 2.494$  MeV. Only one of the three lepton number violating parameters is assumed to be nonvanishing for each of the three cases: (a)  $\langle \lambda \rangle \neq 0$ , (b)  $\langle \eta \rangle \neq 0$  and (c)  $\langle m_{\nu} \rangle \neq 0$ .

Table 1 The operators  $M_{inm}$  and the coefficients  $C_{\mathrm{m}i}$ , the latter in units of the electron-nucleon mass ratio  $m_e/M$ .

i	$oldsymbol{M}_{inm}$	$C_{\mathrm mi}$
1	$-\sqrt{3}h' m{S}_0 m{y}_2$	$-\frac{1}{3}[\mu_{eta}(g_{ m V}/g_{ m A})+\frac{1}{2}]$
2	$h'oldsymbol{S}_2$	$rac{1}{6}[\mu_{eta}(g_{ m V}/g_{ m A})-1]$
3	$h'[oldsymbol{S}_2\otimesoldsymbol{y}_2]^{(2)}$	$-rac{\sqrt{7}}{4\sqrt{3}}[\mu_{eta}(g_{\mathrm{V}}/g_{\mathrm{A}})-1]$
4	$h'oldsymbol{y}_2$	$rac{1}{2}(g_{ m V}/g_{ m A})^2$
5	$h'[oldsymbol{S}_+\otimesoldsymbol{y}_2]^{(2)}$	$rac{\sqrt{3}}{4\sqrt{2}} [\mu_{eta} (g_{ m V}/g_{ m A})^2 - (g_{ m V}/g_{ m A})]$
14	$-\sqrt{3}hm{S}_0m{y}_2'$	$\frac{1}{3}$
15	$holdsymbol{y}_2'$	$-(g_{ m V}/g_{ m A})^2$
16	$Holdsymbol{S}_2$	$-rac{1}{2}[\mu_eta(g_{ m V}/g_{ m A})-1]$
17	$holdsymbol{S}_2oldsymbol{y}_0'$	$-\frac{1}{\sqrt{3}}$
18	$h[oldsymbol{S}_2\otimesoldsymbol{y}_1']^{(2)}$	$-\frac{\sqrt{3}}{2}$
19	$h[oldsymbol{S}_2\otimesoldsymbol{y}_2']^{(2)}$	$-rac{\sqrt{7}}{2\sqrt{3}}$
20	$h[oldsymbol{S}_+\otimesoldsymbol{y}_1']^{(2)}$	$rac{1}{2\sqrt{2}}(g_{ m V}/g_{ m A})$
21	$h[oldsymbol{S}_+\otimesoldsymbol{y}_2']^{(2)}$	$rac{\sqrt{3}}{2\sqrt{2}}(g_{ m V}/g_{ m A})$
22	$h[oldsymbol{S}_1\otimesoldsymbol{Y}_1']^{(2)}$	$-\frac{1}{4}$
23	$h[{m S}_1\otimes {m Y}_2']^{(2)}$	$-\frac{\sqrt{3}}{4}$
24	$h[oldsymbol{S}_{-}\otimesoldsymbol{Y}_{1}^{\prime}]^{(2)}$	$-rac{1}{4\sqrt{2}}(g_{ m V}/g_{ m A})$
25	$h[oldsymbol{S}\otimes oldsymbol{Y}_2']^{(2)}$	$-rac{\sqrt{3}}{4\sqrt{2}}(g_{ m V}/g_{ m A})$

Table 2 Calculated matrix elements  $M_{\rm m}$  for the  $^{76}{\rm Ge}$  and  $^{100}{\rm Mo}$  decays. The entries are the values of the products  $C_{{\rm m}i}M_i$  and their sum  $M_{\rm m}$  in units of  $10^{-3}{\rm fm}^{-1}$ .

	•	1110
i	$^{76}\mathrm{Ge}$	$^{100}\mathrm{Mo}$
1	-0.0229	-0.0077
2	0.0013	0.0003
3	0.0002	0.0008
4	-0.0017	-0.0011
5	-0.0003	0.0007
14	-0.0191	-0.0227
15	-0.0128	-0.0112
16	-0.0033	-0.0006
17	-0.0017	0.0000
18	-0.0028	0.0019
19	-0.0005	0.0001
20	0.0006	0.0005
21	0.0002	-0.0001
22	0.0020	-0.0000
23	0.0020	-0.0026
24	-0.0047	-0.0001
25	0.0012	-0.0010
sum	-0.0624	-0.0427

Table 3 Calculated matrix elements for the  $0^+ \to 2_1^+ \ 0\nu\beta\beta$  decays of  $^{76}\text{Ge}$  and  $^{100}\text{Mo}$  in units of  $10^{-3}\text{fm}^{-1}$ .

	$M_{\lambda}$	$M_{\eta}$	$M'_{\eta}$	$M_{ m m}$
$^{76}\mathrm{Ge}$	1.81 <sup>a</sup>	13.37 a	0.18 <sup>a</sup>	-0.0624
$^{100}\mathrm{Mo}$	-6.33	3.38	5.17	-0.0427

<sup>&</sup>lt;sup>a</sup> Ref. [11].

Table 4 Constraints on the right-handed current couplings and the effective neutrino mass.

	$^{76}\mathrm{Ge}$	$^{100}\mathrm{Mo}$
$ \langle \lambda  angle $	$<8.9\times10^{-4}$	$<3.9\times10^{-4}$
$ \langle \eta  angle $	$<1.2\times10^{-4}$	$< 4.3 \times 10^{-4}$ a
$ \langle m_{\nu} \rangle  \text{ [eV]}$	$<1.0\times10^4$	$<2.2\times10^{4~\rm a}$

<sup>&</sup>lt;sup>a</sup> Assuming the same limit on  $\tau_{1/2}^{0\nu}$  as the  $\langle\lambda\rangle$  mode.